

# MeTEOR Performance Task

## Algebra II

Mathematics

Volume of the Coffee Mug Container

## **Performance Task Item: Volume of the Coffee Mug Container**

Grade Level: Algebra 2

**Focus Area:** Long Division and Remainder Theorem

**Essential Question:** What does the degree of a polynomial tell you about its related polynomial function?

**Core Ideas:**

- Understands the long division process.
- Understands factors of polynomials.
- Understands functions can be represented in a variety of ways.
- Understands the order of arranging terms when dividing polynomials.

**Learning Targets:**

- Students will divide use polynomial long division to divide a polynomial by a polynomial.
- Students will rewrite rational expressions as the quotient in the form of a polynomial added to the remainder divided by the divisor.
- Students will use the Remainder Theorem to determine if  $(x - a)$  is a factor of a polynomial.
- Students will use the Remainder Theorem to determine the remainder of  $p(x)/(x - a)$ .
- Students will explain their reasoning.

### **STANDARDS**

#### **Domain: Algebra: Arithmetic With Polynomials & Rational Expressions**

**Content Standards:**

- Know and apply the Remainder Theorem. For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**Supporting Standards:**

- Rewrite rational expressions.
- Know precise definitions of polynomial terminology.

**Math Practice Standards:**

MP 1: Make sense of problems and persevere in solving them.

MP 2: Reason abstractly and quantitatively.

MP 3: Construct viable arguments and critique the reasoning of others.

MP 4: Model with mathematics.

MP 6: Attend to precision.

MP 7: Look for and make use of structure.

**Materials:**

- Performance Task
- Pencil
- Paper
- Calculator

**Task/Question 1:**

**DOK Level 1:** Recall & Reproduction

**Math Practice Standard:**

- MP 6: Attend to precision.

**A.** In what order should you arrange the terms when you divide polynomials?

**B.** How do you determine where to place the first term in the quotient?

**C.** How do you know when you are finished dividing a polynomial?

**D.** Divide each using long division:

$$(x^2 - 3x - 40) \div (x + 5)$$


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$$(3x^2 + 7x - 20) \div (x + 4)$$


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$$(x^3 + 3x^2 - x + 2) \div (x - 1)$$


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$$(4x^3 + 21x^2 - x - 24) \div (x + 5)$$


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**E.** Scientists have found that not all eyes are spherical on animals.

Variations in the structure of the eye have happened throughout evolutionary history. Pupils also come in various types, depending on the purpose it may hold for the animal that contains it. Those who need to be active during all times of day and night are better equipped with pupils which can easily adjust, such as the rectangular eye. Sheep, Goats, Octopuses and Toads have these rectangular shaped pupils.

If the area of a goat's pupil is  $(x^3 - x^2 - 4x + 4)$  and one of its side lengths is  $(x - 2)$ , what is the other dimension of the pupil?

**Task/Question 2:**

**DOK Level 2:** Basic Application of Skills and Concepts

**Math Practice Standards:**

- MP 1: Make sense of problems and persevere in solving them.
- MP 6: Attend to precision.

**A.** Why would one choose long division over synthetic division when dividing polynomials?

**B.** Define the Remainder Theorem:

**C.** Use synthetic division and the Remainder Theorem to find  $P(x)$ :

$$P(x) = x^3 + 4x^2 - 8x - 6; a = -2$$

$$P(x) = 2x^4 + 6x^3 + 5x^2 - 45; a = -3$$

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**D.** When a polynomial is divided by  $(x + 4)$ , the quotient is  $(2x + 3)$  with a remainder of 2. Find the polynomial.

**E.** Explain how you got your answer in Part D:

**Task/Question 3:**

**DOK Level 3:** Strategic Thinking and Complex Reasoning

**Math Practice Standards:**

- MP 1: Make sense of problems and persevere in solving them.
- MP 2: Reason abstractly and quantitatively.
- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 4: Model with mathematics.
- MP 7: Look for and make use of structure.

The total number of visitors who went to Tropic’s Theme Park from December 1 to December 10 can be modeled by the function  $F(x) = 126x^3 + 279x^2 + 90x + 450$ .

The number of shows played at the theme park from December 1 to December 10 can be modeled by  $G(x) = 18x + 45$ , where  $x$  is the number of days since December 1.

- A. Write the expression that describes the average number of visitors per show?
  
- B. If  $x = 7$ , what is the average number of visitors per show?
  
- C. Tropic’s Theme Park decides to promote the shows to get an average of 500 people per show. What is the minimum value that  $x$  can be in order to have at least 500 per show?
  
- D. If your expression in Part A had a remainder of 58, what would the original function  $F(x)$  have been?
  
- E. Explain how you determined the answer to Part D and how it would affect the minimum value in Part C. Justify and defend how your approach to solving this is the **most efficient**.

**Task/Question 4:**

**DOK Level 3:** Strategic Thinking and Complex Reasoning

**Math Practice Standards:**

- MP 1: Make sense of problems and persevere in solving them.
- MP 2: Reason abstractly and quantitatively.
- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 4: Model with mathematics.
- MP 7: Look for and make use of structure.

**Betty's Coffee Shop** sells coffee mugs in her coffee shop. Each mug comes in a box for safe keeping. The polynomial  $(x^3 + 5x^2 - 9x - 45)$  expresses the volume, in cubic inches, of a box used to hold the coffee mug. The height of the box is  $(x + 5)$ .

- A. What are the other dimensions of the box?
  
- B. Customers like the mugs so much that Betty has started selling them by the carton for offices to hand out as gifts. The carton needed to fill this request only comes in one size. The dimensions of the carton are double the dimensions of the coffee mugs box. What are the dimensions of the carton?
  
- C. What is the volume of the carton in Part B?
  
- D. How many mugs fit in the carton from Part C?
  
- E. Explain how you determined the answer to Part D. Justify and defend how your approach to solving this is the **most efficient**.

## Complete Performance Task Scoring Rubric *Volume of the Coffee Mug Container*

22-24 Proficient   19-21 Good   17-18 Satisfactory   14-16 Poor   0-13 Unsatisfactory

	Depth of Knowledge Level	Points	Total Possible Points for Task	Total Points Earned by Student
<p><b>Task 1:</b></p> <p>A. Descending</p> <p>B. It is placed over the term in the dividend with the same degree.</p> <p>C. The degree of the remained is less than the divisor.</p> <p>D. <math>x - 8</math>; <math>3x - 5</math> <math>x^2 + 4x + 3</math> R 5; <math>4x^2 + x - 6</math> R 6</p> <p>E. <math>x^2 + x - 2</math></p>	<b>1</b>	<b>1</b>  <b>1</b>  <b>1</b>  <b>2</b>  <b>1</b>	<b>6</b>	
<p><b>Task 2:</b></p> <p>A. Long Division works on all division problems. Synthetic Division only can be used when you divide by a linear expression.</p> <p>B. If you divide a polynomial <math>P(x)</math> of degree <math>n \geq 1</math> by <math>x - a</math>, then the remainder is <math>P(a)</math></p> <p>C. 18, 0</p> <p>D. <math>2x^2 + 11x + 14</math></p> <p>E. Answers will vary. Possible Explanation: "I multiplied the polynomials and added 2 to the result."</p>	<b>2</b>	<b>2</b>  <b>1</b>  <b>1</b>  <b>1</b>	<b>6</b>	



<p><b>Task 3:</b></p> <p>A. <math>7x^2 - 2x + 10</math></p> <p>B. 339</p> <p>C. 9</p> <p>D. <math>126x^3 + 279x^2 + 90x + 508</math></p> <p>E. Answers may vary. Possible Explanation: “I knew that when <math>x</math> was equal to 7, there were not enough people at the show. When I changed 7 to 8 and multiplied it out, it still was under the 500 minimum the company wanted. Substituting 9 in gave me the minimum, but was over the 500. I decided to go back and work with the value of <math>x</math> being equal to 8. To have it equal 500, I had to add in the difference which was the 58.”</p>	<b>2</b>	<b>1</b>  <b>1</b>  <b>1</b>  <b>1</b>  <b>2</b>	<b>6</b>	
<p><b>Task 4:</b></p> <p>A. <math>(x - 3)</math> inches, <math>(x + 3)</math> inches</p> <p>B. <math>(2x + 10)</math> inches, <math>(2x - 6)</math> inches, <math>(2x + 6)</math> inches</p> <p>C. <math>8x^3 + 40x^2 - 72x - 360</math> cubic inches</p> <p>D. 8</p> <p>E. Answers may vary. Possible Explanation: “First, I doubled the dimensions and multiplied out the new volume. I noticed the leading coefficient was 8 times the original number as were each of the other coefficients. To check my work, I drew a picture to show the dimensions doubled and stacked the coffee mugs to model the problem. There were 8.”</p>	<b>3</b>	<b>1</b>  <b>1</b>  <b>1</b>  <b>1</b>  <b>2</b>	<b>6</b>	
<b>TOTAL POINTS:</b>				



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